ELECTROMAGNETIC INDUCTION

01/08/2023

FARADAY'S LAWS

When φ_B through a loop changes, an emf is induced in the loop which lasts as long as the φ_B changes.
(magnetic flux)

$$\boxed{\epsilon = \frac{-d\varphi_B}{dt}}$$

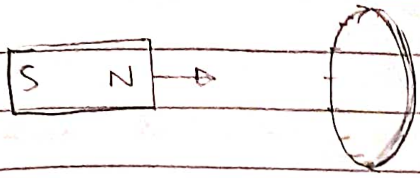
where $\boxed{\varphi_B = \int_S \vec{B} \cdot d\vec{s}}$

NOTE: $\oint_S \vec{B} \cdot d\vec{s} = 0 \Rightarrow$ Magnetic flux through a closed surface is ZERO

$\Rightarrow \nexists$ Magnetic monopole

REMARK: $-$ sign in FL, indicates the emf produced opposes change of φ_B

- Lenz's law $-$ Induced current in a loop will oppose the very cause responsible for inducing current.

eg- 

If N is moved towards the fixed loop,

$\frac{d\varphi_B}{dt} \uparrow$

To oppose this change, current is induced s.t $\frac{d\varphi_B}{dt} \downarrow$. i.e face of loop on magnet's side would act like N pole.

Hence anti clockwise current is induced.

Since $\varphi_B = \vec{B} \cdot \vec{A}$

$= B \cdot A \cos\theta$

To change φ_B, we can change any of the 3 qty.

Motional emf - A or $\cos\theta$ changes

Time varying B - B changes

Motional emf is induced only in conductors, whereas emf is induced in space itself in case of time varying B.

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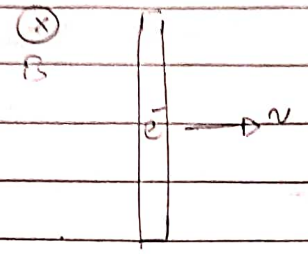
Reason : Cause of motional emf is the force on free e^-, which causes change in charge distribution & development of $\vec{E}$, hence inducing emf.

eg -

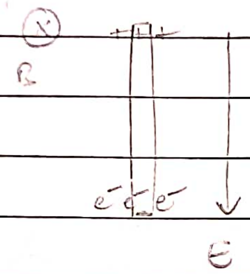


Due to motion of rod, e^- acquires vel. v.

So, it experiences a force $e(\vec{v} \times \vec{B})$

$\Downarrow$

Accumulation of charges creates E.



In eq.

$$-e\vec{E} + (-e)(\vec{v} \times \vec{B}) = 0$$

$$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$$

This V acts as induced emf in conductors.

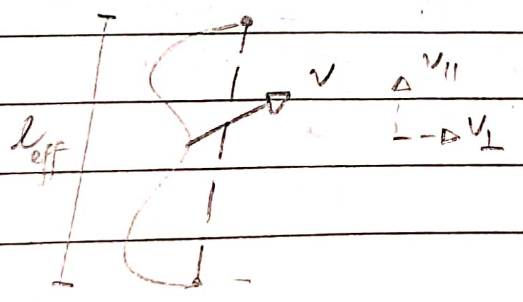
$$dV = -\vec{E} \cdot d\vec{l}$$

$$\Rightarrow \boxed{V = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}}$$

When B, v, l are mutually $\perp$,

$$\boxed{\varepsilon = Bvl}$$

eg -



$$\boxed{\varepsilon = B_{\perp} v_{\perp} l_{eff}}$$

$B \perp$ to plane of motion

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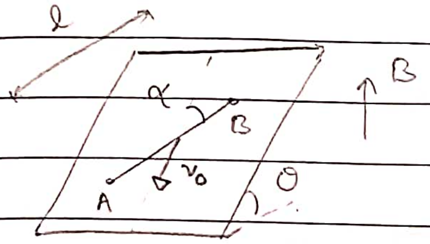
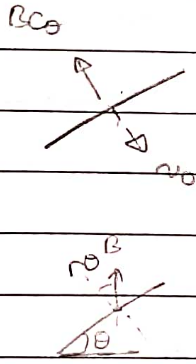
NOTE: if $\left.\begin{matrix}\vec{B} \parallel \vec{v} \\ \vec{B} \parallel \vec{l} \\ \vec{v} \parallel \vec{l}\end{matrix}\right\} \Rightarrow \varepsilon = 0$ (property of scalar triple product)

Q. 

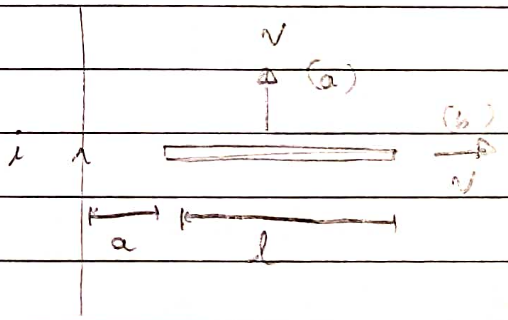
Find ε & end which is a greater potential

A. 

$\varepsilon = (B\cos\theta)(v\sin\alpha)(l)$

$= \underline{Bvl \sin\alpha \cos\theta}$

A is at greater potential.

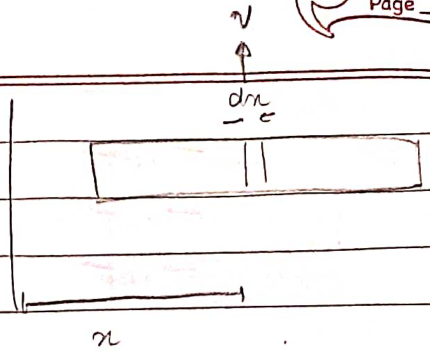
Q. 

Find ε in cases (a) & (b)

A. b) $\varepsilon = 0$ since $\vec{v} \parallel \vec{l}$

a) $\vec{v} \parallel \vec{B} \parallel l$ but B is diff. for diff. pts.

$$B_x = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{i}{x}\right)$$

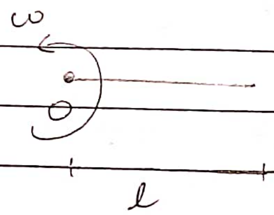


$$d\varepsilon = B_x v\, dx$$

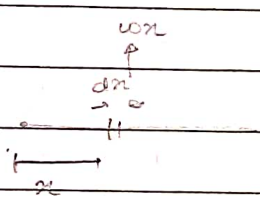
$$= \left(\frac{\mu_0 i v}{2\pi}\right)\left(\frac{dx}{x}\right)$$

$$\varepsilon = \int_{x=a}^{x=(a+l)} d\varepsilon = \frac{\mu_0 i v}{2\pi}\, l\left(1+\frac{l}{a}\right)$$

Q. ⊗ B Find ε.



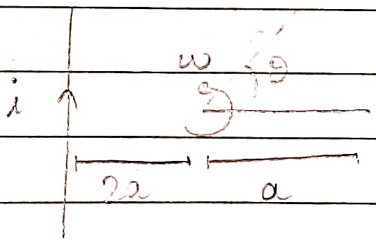
A.



$$d\varepsilon = Bv\, dx = B\omega x\, dx$$

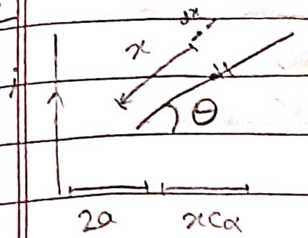
$$\varepsilon = \int_{x=0}^{x=l} d\varepsilon = \frac{B\omega l^2}{2}$$

Q.



Find ε when rod at θ

A.



$$d\epsilon = B_x v_x\, dx$$
$$= \left(\frac{\mu_0 i}{2\pi}\right)\left(\frac{1}{2a + x C_\theta}\right)(\omega x)\, dx$$

$$\epsilon = \int_{x=0}^{x=a} d\epsilon = \frac{\mu_0 i \omega}{2\pi C_\theta}\int_0^a \left(1 - \frac{2a}{2a + x C_\theta}\right) dx$$
$$= \left(\frac{\mu_0 i \omega}{2\pi C_\theta}\right)\left(a - \frac{2a}{C_\theta}\,\ell\left(1 + \frac{C_\theta}{2}\right)\right)$$

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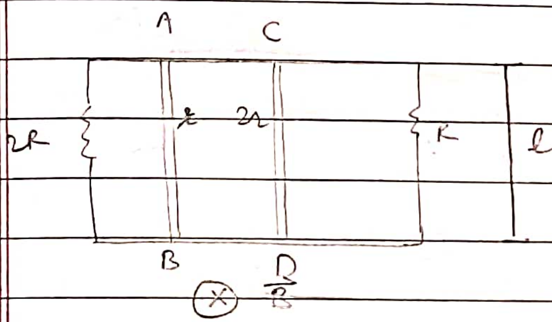
A conductor moving in $\vec{B}$ will act as battery for a circuit

Q.



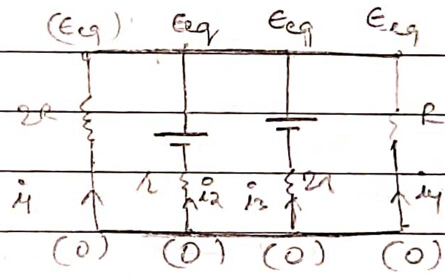
Find current through each resistor & rod if both rods moving with same speed v

a) In same dirn

b) In opp dirn

A. a) $E_{AC} = Bvl$

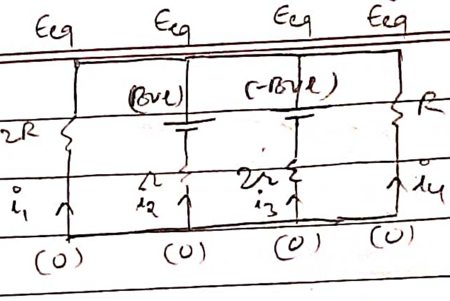
$E_{CD} = Bvl$



$$E_{eq} = \frac{\frac{Bvl}{r} + \frac{Bvl}{2r}}{\frac{1}{r} + \frac{1}{2r} + \frac{1}{R} + \frac{1}{2R}} = \frac{\frac{3Bvl}{2r}}{\frac{3R + 3r}{2Rr}}$$
$$= \left(\frac{BvlR}{r + R}\right)$$

$i_1 = \frac{Bvl}{2(r+R)}$ $\quad i_2 = \frac{\frac{BvlR}{r+R} - Bvl}{r} = -\left(\frac{Bvl}{r+R}\right)$ $\quad i_3 = \frac{-Bvl}{2(r+R)}$ $\quad i_4 = \frac{Bvl}{(r+R)}$

b) $\mathcal{E}_{eq} = \dfrac{\dfrac{Bvl}{r} - \dfrac{Bvl}{2r}}{\dfrac{1}{r}+\dfrac{1}{2r}+\dfrac{1}{R}+\dfrac{1}{2R}}$

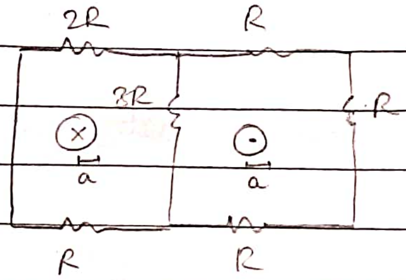


$= \dfrac{BvlR}{3(r+R)}$

$i_1 = \dfrac{Bvl}{6(r+R)}$ $\quad i_2 = \dfrac{-Bvl(3r-2R)}{3(r+R)}$ $\quad i_3 = \dfrac{-Bvl(3r+4R)}{6(r+R)}$

$i_4 = \dfrac{Bvl}{3(r+R)}$

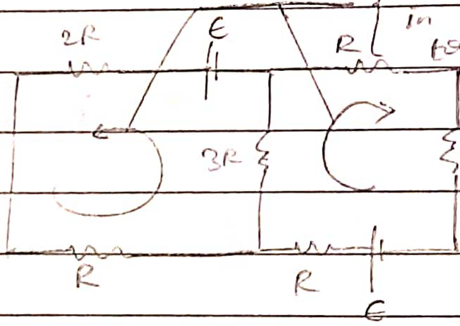
Q



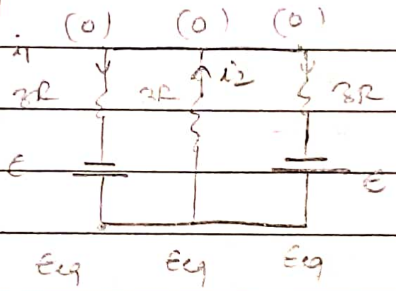
$\dfrac{dB}{dt} = \alpha$ in both regions.

Find current through each resistor

A. $\dfrac{d\varphi}{dt} = \pi a^2 \alpha \implies$



{Current flowing in opp. dirn to oppose $\dfrac{d\varphi}{dt}$}



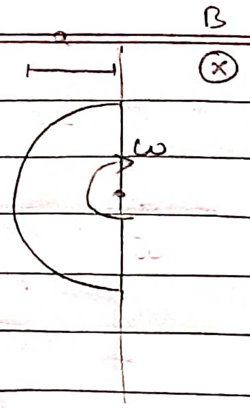
{* We can represent induced emf with a battery anywhere in circuit except the common side}

$\mathcal{E}_{eq} = \dfrac{\dfrac{E}{2R} + \dfrac{E}{3R}}{\dfrac{1}{2R}+\dfrac{1}{3R}+\dfrac{1}{2R}} = \left(\dfrac{2E}{3}\right)$

$i_1 = i_3 = \dfrac{\epsilon}{9R} = \left(\dfrac{\pi a^2\alpha}{9R}\right)$

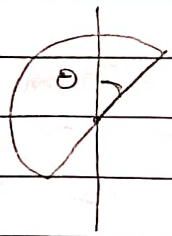
$i_2 = \dfrac{2E}{9R} = \dfrac{2\pi a^2\alpha}{9R}$

Q.



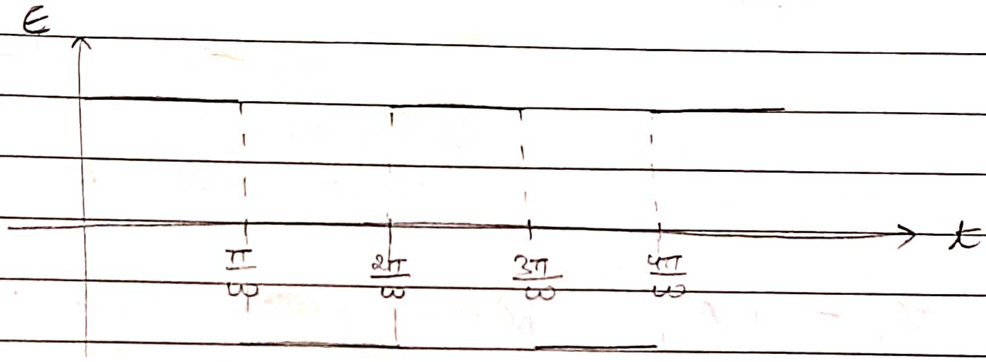
Find $\mathcal{E}_{induced}$ in the loop as a fn of time.
Also draw the graph b/w induced emf & time.

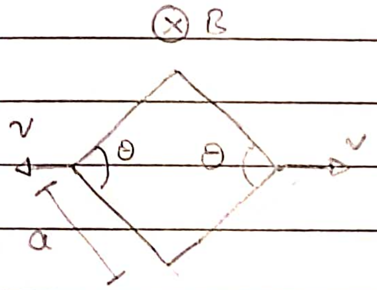
A.



$$A = \frac{a^2}{2}\theta = \left(\frac{a^2\omega t}{2}\right)$$

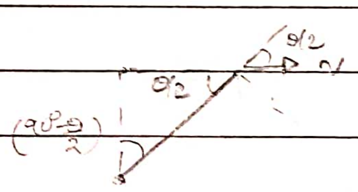
$$\frac{d\varphi}{dt} = \frac{d}{dt}(BA) = \frac{Ba^2}{2}\left(\frac{d\theta}{dt}\right) = \left(\frac{Ba^2\omega}{2}\right)$$





Q. Find $\mathcal{E}_{induced}$ when θ reduces to 60°.

A.



$$A = 2\left(a\cos\frac{\theta}{2}\right)\left(a\sin\frac{\theta}{2}\right) = a^2\sin\theta$$

$$v = \frac{d}{dt}(a\cos\theta/2) = -\frac{a}{2}\sin\left(\frac{\theta}{2}\right)\left(\frac{d\theta}{dt}\right)$$

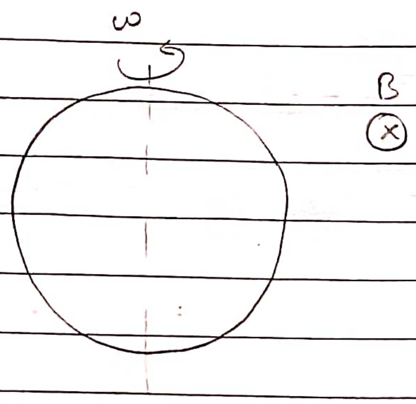
$$\frac{d\varphi}{dt} = \frac{d(B\cdot a^2\sin\theta)}{dt} = a^2B\cos\theta\,\frac{d\theta}{dt}$$

$$= -2avB\,\operatorname{cosec}\left(\frac{\theta}{2}\right)\cos(\theta)$$

- Total charge flowing through loop –

$$i = \frac{\epsilon}{R} \Rightarrow \frac{dq}{dt} = \frac{d\varphi}{R\,dt} \Rightarrow \boxed{q = \int \frac{d\varphi}{R}}$$

$$= \frac{\Delta\varphi}{R}$$

Q. 

Find net charge that flows through the ring if it rotates through an angle

a) $\pi/2$ b) π c) 2π

about one of its diameters.

A. $\varphi_i = BA$ (area vector assumed in dirn of B)

a) $\varphi_f = 0 \Rightarrow q = -\left(\frac{BA}{R}\right)$

b) $\varphi_f = -BA \Rightarrow q = -\left(\frac{2BA}{R}\right)$

c) $\varphi_f = BA \Rightarrow q = 0$

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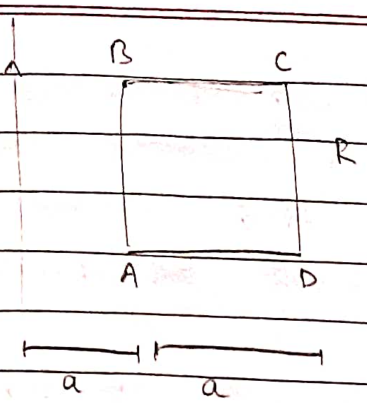
Q.

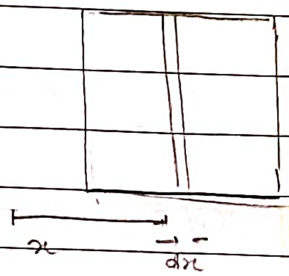


Find charge that flows through the sq. loop if it is turned about AB through

a) $90°$ b) $120°$

A. $\varphi_i = \int d\varphi = \int_a^{2a} \left(\frac{\mu_0}{2\pi}\right)\left(\frac{i}{x}\right) a\,dx$

$= \frac{\mu_0 i a}{2\pi} \ell(2)$

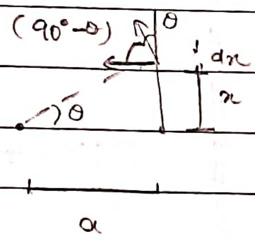


a) $\varphi_f = \int d\varphi = \int B\,(a\,dx)\cos\theta$

$= \int_0^a \left(\frac{\mu_0}{2\pi}\right)\left(\frac{i a}{\sqrt{x^2+a^2}}\right) \frac{x}{\sqrt{x^2+a^2}}\, dx$

$= \frac{\mu_0 i a}{4\pi}\left[\ell(x^2+a^2)\right]_0^a$

$= \frac{\mu_0 i a\, \ell(2)}{4\pi}$

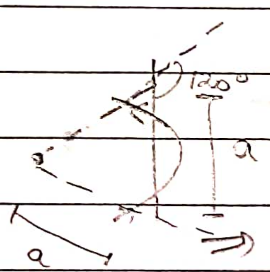


$x = a\tan\theta$

$\Rightarrow dx = a\sec^2(\theta)\,d\theta$

$\Rightarrow q = \frac{\varphi_f - \varphi_i}{R} = -\frac{\mu_0 i a \ell(2)}{4\pi R}$

b)



By sym., $\varphi_f = 0$

$\Rightarrow q = \frac{\varphi_f - \varphi_i}{R} = -\frac{\mu_0 i a \ell(2)}{2\pi R}$

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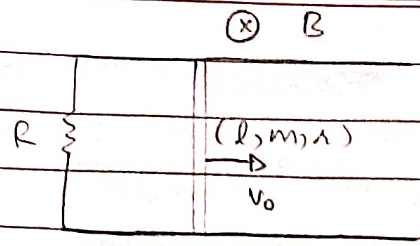
Q



The rod is pulled || to rails with const. vel. v_0, then find

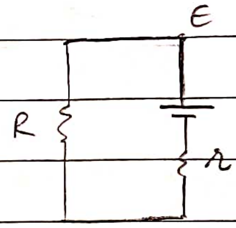
a) current in circuit

b) V across rod

c) Force req. to move the rod with const. vel.

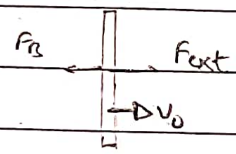
A

(i) $\epsilon = Bv_0 l \Rightarrow i = \left(\frac{Bvl}{R+r}\right)$



(ii) $V = \epsilon - ir = \frac{(Bvl)(R)}{(R+r)}$

(iii)



$F_B = ilB$

$= \frac{B^2vl^2}{(R+r)}$

$\therefore F_{ext} = F_B$

$= \frac{B^2vl^2}{(r+R)}$

Q In the above Q, if rod is projected with vel. v_0 || to the rails, then find vel. of rod.

a) as a fxn of time

b) as a fxn of dist covered

A. $F_B = \frac{B^2l^2}{(R+r)}v \Rightarrow a = -\frac{B^2l^2}{(R+r)m}v$

$\Rightarrow \int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\frac{B^2l^2}{(R+r)m}\,dt$

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$$\Rightarrow \quad v = v_0 e^{-\frac{B^2 l^2}{m(R+r)} t}$$

$$\int_0^x dx = \int_0^t v_0 e^{-\frac{B^2 l^2}{m(R+r)} t} \, dt$$

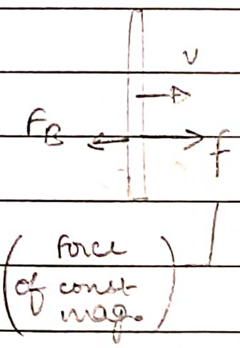
$$\Rightarrow \quad x = \frac{v_0 m(R+r)}{B^2 l^2} \left[1 - e^{-\frac{B^2 l^2}{m(R+r)} t} \right]$$

$$\Rightarrow \quad x = \frac{m(R+r)}{B^2 l^2} (v_0 - v)$$

$$\Rightarrow \quad v = v_0 - \frac{B^2 l^2}{m(R+r)} x.$$

Q. In the above Q, if a force of const. mag. is applied on rod at rest. Find v as a f^{n} of time & v_{max}.

A.



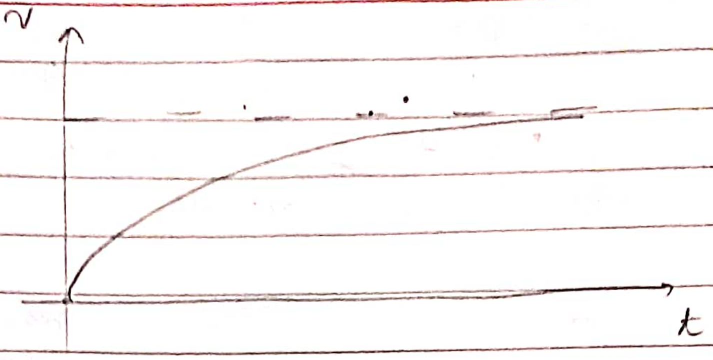
$$a = -\frac{(F_B - f)}{m} = -\frac{B^2 l^2}{m(r+R)} v + \left(\frac{f}{m}\right)$$

$$\Rightarrow \quad \frac{dv}{dt} + \frac{B^2 l^2}{m(r+R)} v = \left(\frac{f}{m}\right)$$

$$\Rightarrow \quad v = \frac{f(r+R)}{B^2 l^2} \left[1 - e^{-\frac{B^2 l^2 t}{f(r+R)}} \right]$$

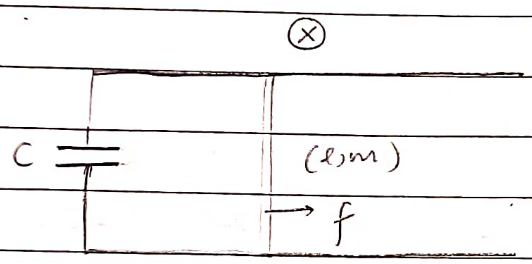
$$v_{max} = \frac{f(r+R)}{B^2 l^2}$$

at v_{max} $\quad F_B = f$



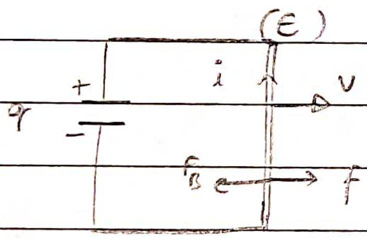
$$v = v_{max}\left(1 - e^{-\lambda t}\right)$$

$$\lambda = \frac{B^2 l^2}{f(R+r)}$$

Q. 

Resistance less circuit. Conducting rod initially at rest.

Find v of rod & charge on cap. as a f^n of time.

A. 

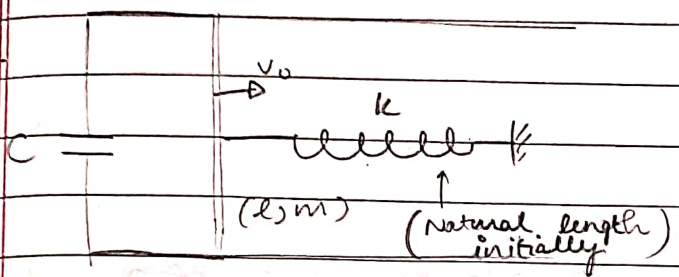
$$q = EC = (BlC)\,v$$

$$i = \frac{dq}{dt} = (BlC)\left(\frac{dv}{dt}\right) = (BlC)(-a)$$

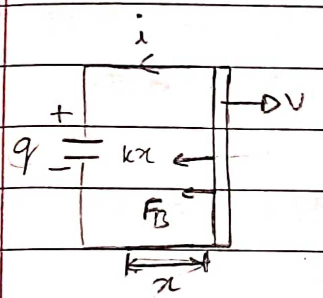
$$a = -\frac{(F_B - f)}{m} = -\frac{ilB}{m} + \frac{f}{m} = -\frac{B^2l^2C\,a}{m} + \frac{f}{m}$$

$$\Rightarrow \quad a = \left(\frac{f}{m + B^2l^2C}\right) \quad \Rightarrow \quad v = \left(\frac{ft}{m + B^2l^2C}\right)$$

NOTE: If instead, rod was projected with v_0 & no ext force was applied, the cap. would charge instant ⇒ no current flow in circuit
⇒ rod moves unaffected.

Q. 

P.T rod will exhibit SHM.
Find T & A of SHM.

A 

$$q = EC = (BlC)v$$

$$\Rightarrow i = \frac{dq}{dt} = (BlC)\left(\frac{dv}{dt}\right) = (BlC)(-a)$$

$$a = -\frac{(F_B + kx)}{m} = -\frac{ilB}{m} - \frac{k}{m}x$$

$$\Rightarrow a\left(1 + \frac{B^2l^2C}{m}\right) = -\frac{kx}{m}$$

$$\Rightarrow \ddot{x} = -\left(\frac{k}{m+B^2l^2C}\right)x$$

$$T = 2\pi\sqrt{\frac{m+B^2l^2C}{k}}$$

$$v_0 = A\omega = A\sqrt{\frac{k}{m+B^2l^2C}}$$

$$\Rightarrow A = v_0\sqrt{\frac{m+B^2l^2C}{k}}$$

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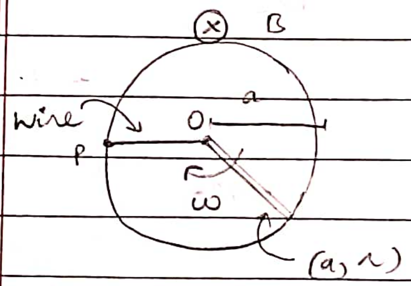
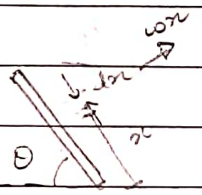
Energy Method

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 + \frac{1}{2}C\underbrace{V^2}_{(Bvl)^2} = \text{const.}$$

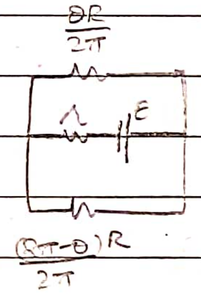
$$\Rightarrow \left(\frac{m+B^2l^2C}{2}\right)v^2 + \frac{k}{2}x^2 = \text{const.} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m+B^2l^2C}{k}}$$

Q. 

Resistance of ring is R. Find current in rod as a fr^n of angle b/w rod & OP.
Also find the torque of Lenz force about the pivot.

A. 

$$\mathcal{E} = \int Bv\,dx = \int_0^a B\omega x\,dx = \frac{B\omega a^2}{2}$$



$$E_{eq} = \frac{\frac{E}{r}}{\frac{1}{r} + \frac{2\pi}{\theta R} + \frac{2\pi}{(2\pi-\theta)R}} = \frac{ER}{R + \frac{4\pi^2 r}{\theta(2\pi-\theta)}}$$

$$i = \frac{E - E_{eq}}{r} = \frac{4\pi^2}{\theta(2\pi-\theta)}\left(\frac{E}{R+\left(\frac{4\pi^2 r}{\theta(2\pi-\theta)}\right)}\right)$$

$$= \frac{B\omega a^2}{\left(\frac{\theta(2\pi-\theta)R + 2r}{2\pi^2}\right)}$$

$$i_{max} = \frac{B\omega a^2}{2r} \quad (\theta = 0) \qquad i_{min} = \left(\frac{2B\omega a^2}{R+4r}\right) \quad (\theta = \pi)$$

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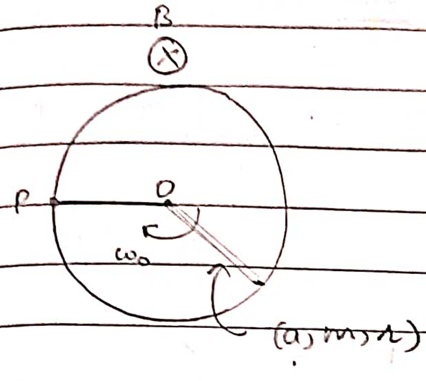
* only when line of action of force $\perp$ rod

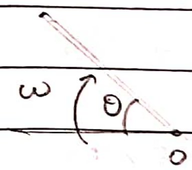
$$\int_0^{\tau} d\tau = \int_0^{a} (iB\,dx)(x) \Rightarrow \tau = \frac{Bia^2}{2} = (Bia)\left(\frac{a}{2}\right)^{*}$$

F_B (dist of centre)



Q. Resistance less rim.
Rod is projected with ω_0.
Find ω of rod as a fn^n of time & angle rotated

A



$$E = \int B\omega x\,dx = \frac{B\omega a^2}{2}$$

$$i = \frac{E}{r} = \frac{B\omega a^2}{2r}$$

$$\tau = \left(-\frac{Ba^2}{2}\right) i = \left(-\frac{B^2a^4}{4r}\right)\omega \Rightarrow \left(\frac{ml^2}{3}\right)(\alpha) = \left(-\frac{B^2a^4}{4r}\right)\omega$$

$$\Rightarrow \int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = \int_0^t \frac{-3B^2a^4}{4ml^2r}\,dt$$

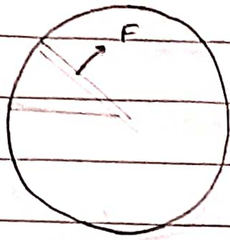
$$\Rightarrow \omega = \omega_0 e^{\frac{-3B^2a^4 t}{4ml^2r}}$$

$$\Rightarrow \int_0^{\theta} d\theta = \int_0^t \omega_0 e^{-\lambda}\,dt \Rightarrow \theta = \frac{\omega_0}{\lambda} e^{-\lambda t}$$

$$\Rightarrow \omega = \lambda\theta \qquad \lambda = -\frac{3B^2a^4}{4ml^2r}$$

Q. In the above Q, if const. F applied at centre, find ω as a f^n of t.

A.



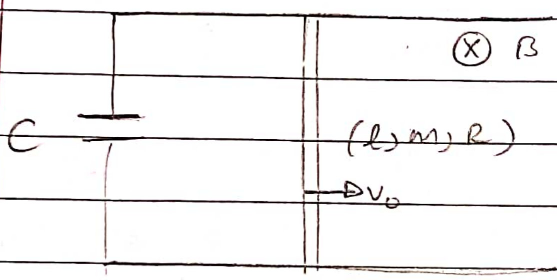
$$\tau = \frac{Fa}{2} - \left(\frac{B^2a^4}{4r}\right)\omega$$

$$\Rightarrow \left(\frac{ml^2}{3}\right)\left(\frac{d\omega}{dt}\right) + \left(\frac{B^2a^4}{4r}\right)\omega = \left(\frac{Fa}{2}\right)$$

$$\tau_0 = \left(\frac{Fa}{2}\right)$$

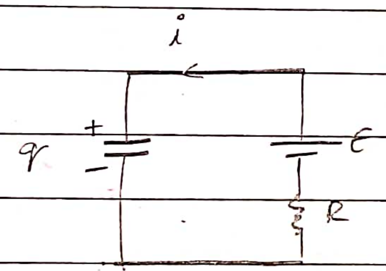
$$\omega = \frac{2rF}{B^2a^3}\left(1 - e^{-\frac{3B^2a^4t}{4rml^2}}\right)$$

Q.



Find charge on cap., current in circuit & force req. to move rod with const. vel. as a f^n of time.

A.



$$-\frac{q}{C} - iR + \epsilon = 0$$

$$\Rightarrow R\frac{dq}{dt} + \frac{q}{C} = \epsilon$$

$$\Rightarrow q = \epsilon C\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i = \frac{\epsilon}{R}e^{-t/RC}, \qquad \epsilon = Bv_0l$$

$$F = iBl = \frac{\epsilon Bl}{R}e^{-t/RC}$$

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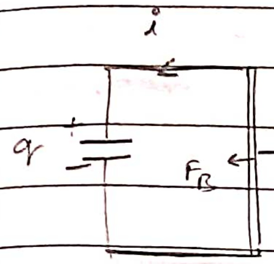
Q. In the above Q, if rod projected with v_0, find charge on cap. & current in circuit as a fn of time.

A.



$$-\frac{q}{C} - iR + \epsilon = 0$$

$$\Rightarrow R\frac{dq}{dt} + \frac{q}{C} = Blv$$

$$\Rightarrow R\frac{d^2q}{dt^2} + \frac{dq}{C} = Bl\frac{dv}{dt}$$

$$a = \frac{-F_B}{m} = \frac{-ilB}{m}$$

$$\Rightarrow R\frac{d^2q}{dt^2} + \frac{dq}{C} = Bl\left(-\frac{Bl}{m}\right)\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \frac{dv}{dt} = -\left(\frac{Bl}{m}\right)\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \frac{d^2q}{dt^2} = -\left(\frac{1}{C} + \frac{B^2l^2}{m}\right)\left(\frac{1}{R}\right)\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \frac{di}{dt} = -\left(\frac{1}{RC} + \frac{B^2l^2}{Rm}\right) i$$

$$\Rightarrow \int_{i_0}^{i} \frac{di}{i} = \int_0^t -\left(\frac{1}{RC} + \frac{B^2l^2}{Rm}\right) t$$

$$\Rightarrow i = i_0 e^{-\frac{1}{R}\left(\frac{1}{C} + \frac{B^2l^2}{m}\right)t}$$

$$i_0 = \left(\frac{Bv_0l}{R}\right) \qquad \Rightarrow i = \left(\frac{Bv_0l}{R}\right) e^{-\frac{1}{R}\left(\frac{1}{C} + \frac{B^2l^2}{m}\right)t}$$

$$\Rightarrow q = \frac{Bv_0l}{\left(\frac{1}{C} + \frac{B^2l^2}{m}\right)}\left[1 - e^{-\frac{1}{R}\left(\frac{1}{C} + \frac{B^2l^2}{m}\right)t}\right]$$

$$\int_{v_0}^{v} dv = \int_0^q -\frac{Bl}{m}\, dq$$

$$v = v_0 - \frac{Blq}{m}$$

$$v_{terminal} = v_0 - \frac{Bl}{m} q_{max} = v_0\left[1 - \frac{B^2l^2}{\frac{m}{C} + B^2l^2}\right]$$

$$= \left(\frac{mv_0}{m + B^2l^2C}\right)$$

To directly calc. v_T, we could have found v at which $i=0$.

$$\Rightarrow \quad \frac{q}{C} = Bvl \quad \& \quad v = v_0 - \frac{Blq}{m}$$

$$\Rightarrow \quad v = v_0 - \frac{B^2l^2C}{m} v \quad \Rightarrow \quad v = \left(\frac{mv_0}{m+B^2l^2C}\right)$$

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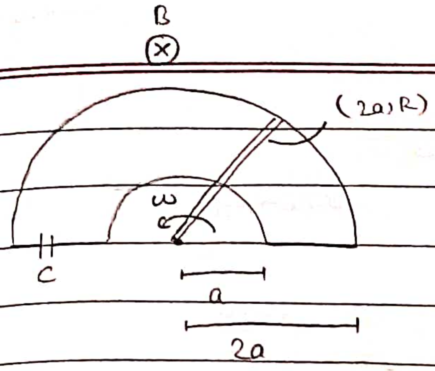
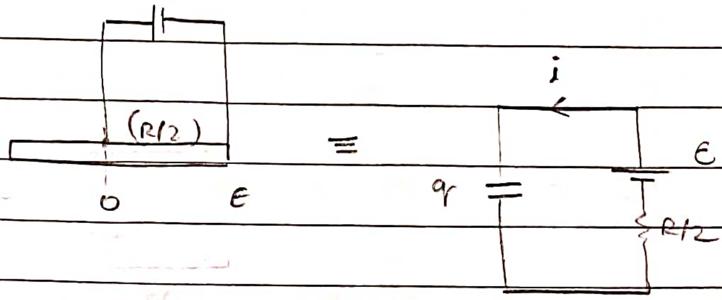


★ Q. Find charge on cap. when rod turns through θ.

A. $\boxed{\epsilon = \left(\frac{B\omega a^2}{2}\right)}$



$$-\frac{q}{C} - \frac{Ri}{2} + \epsilon = 0 \Rightarrow R\frac{dq}{dt} + \frac{2q}{C} = 2\epsilon$$

$$\Rightarrow q = \epsilon C\left(1 - e^{-\frac{2t}{RC}}\right)$$

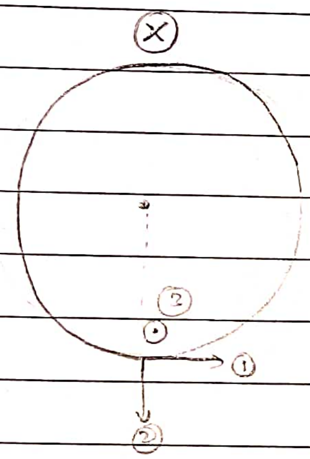
$$\omega t = \theta \Rightarrow q = \epsilon C\left(1 - e^{-\frac{2\theta}{\omega RC}}\right)$$

08/08/2017

INDUCED $\vec{E}$

Produced due to time varying $\vec{B}$ in space

$\vec{E}$	$\vec{E}_{induced}$
- Due to charge	- Due to changing $\vec{B}$
- (+ve) ⟶ (−ve)	- Closed field lines
- Conservative Field (Potential field)	- Non-conservative field
$\Rightarrow \oint_l \vec{E}\cdot d\vec{l} = 0$	$\boxed{\oint_l \vec{E}\cdot d\vec{l} = -\left(\frac{d\varphi}{dt}\right)}$



For determining E_i around a cylindrical $\vec{B}$, we use sym. argument.

We assume E_i has 3 components, 1, 2, 3 at every pt. on the circular path.

$E_3 = 0$ since $E_i \perp B$ (property of E_i)

$E_2 = 0$ since no charge inside cylinderical surface (by Gauss law)

$\Rightarrow E_i = E_1$ i.e $\vec{E_i}$ is along tangent only!

To find dirn of E_i, find dirn of induced current

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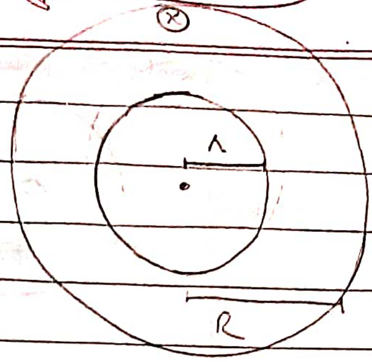
If $r \leq R$

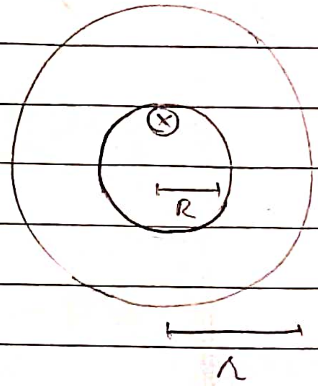


$$\oint E_i \cdot dl = \frac{d\varphi}{dt}$$

$$\Rightarrow (E_i)(2\pi r) = (\pi r^2)\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \boxed{E_i = \left(\frac{r}{2}\right)\left(\frac{dB}{dt}\right)}$$

If $r > R$

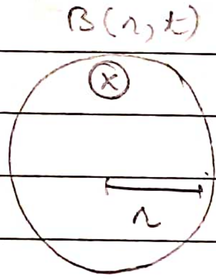


$$\oint E_i \, dl = \frac{d\varphi}{dt}$$

$$\Rightarrow (E_i)(2\pi r) = (\pi R^2)\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \boxed{E_i = \frac{R^2}{2r}\left(\frac{dB}{dt}\right)}$$

If B changing with both r & t



$$\varphi = \int_0^r (2\pi r)(dr)\left(B(r,t)\right)$$

$$\oint E_i \cdot dl = \frac{\partial\varphi}{\partial t}$$

$$\Rightarrow E_i (2\pi r) = \frac{\partial \varphi}{\partial t}$$

$$\Rightarrow \boxed{E_i = \left(\frac{1}{2\pi r}\right)\left(\frac{\partial \varphi}{\partial t}\right)}$$

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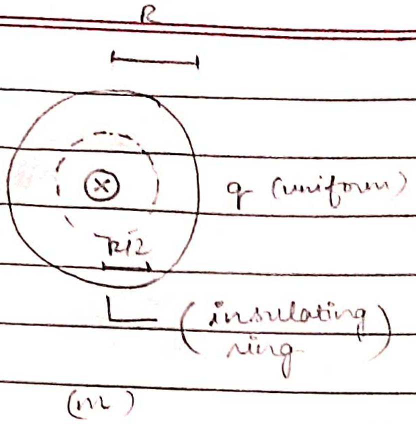
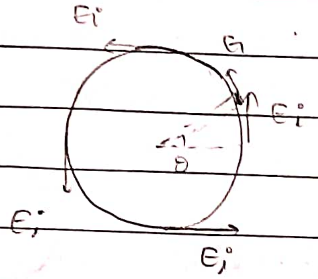
Q.



$\frac{dB}{dt} = \alpha$

Find angular speed of the ring after time 't' if the ring is at rest at $t=0$

A.



$F_{net} = 0$

$d\tau = (E_i\, dq)(R)$

$\Rightarrow \tau = qE_iR$

$(E_i)(2\pi R) = \left(\frac{\pi R^2}{4}\right)(\alpha)$ $\Rightarrow$ $(m\cancel{R^2})\left(\frac{d\omega}{dt}\right) = \frac{q\cancel{R^2}\alpha}{8}$

$\Rightarrow E_i = \left(\frac{R\alpha}{8}\right)$ $\Rightarrow$ $\omega = \left(\frac{q\alpha}{8m}\right)t$

Q In the above Q, if initially there was uniform $\vec{B}$ & it was suddenly switched off, find angular speed acq. by ring.

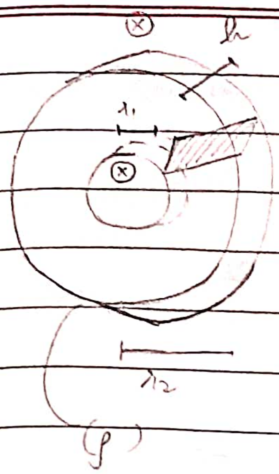
A. $\tau = qE_iR = \left(\frac{qR^2}{8}\right)\left(\frac{dB}{dt}\right)$ $\Rightarrow$ $mR^2\left(\frac{d\omega}{\cancel{dt}}\right) = \left(\frac{qR^2}{8}\right)\left(\frac{dB}{\cancel{dt}}\right)$

$\Rightarrow \int_0^{\omega} mR^2\, d\omega = \int_{B_0}^{0} \frac{qR^2}{8}\, dB$

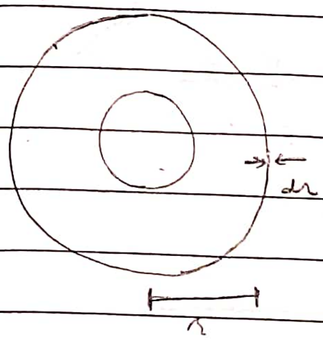
$\Rightarrow \omega = -\frac{qB_0}{8m}$

Q. $\frac{dB}{dt} = \alpha$

Find current inside conductor



A.



$$dR = \frac{\rho (2\pi r)}{h (dr)}$$

$$\varepsilon_r = \pi r^2 \alpha$$

$$di = \frac{\varepsilon_r}{dR} = \frac{\cancel{\pi} r^{\cancel{2}} \alpha \,(h\, dr)}{\rho (2\cancel{\pi}\cancel{r})}$$

$$\Rightarrow \quad i = \frac{h\alpha}{2\rho}\int_{r_1}^{r_2} r\, dr = \underline{\frac{h\alpha (r_2^2 - r_1^2)}{4\rho}}$$

<u>NOTE:</u> If $\vec{B}$ was given only inside, ε for every elem. would have been same. In that case we could have calc. Req. (all elems. in ||) & $\underline{i = \varepsilon / R_{eq}}$

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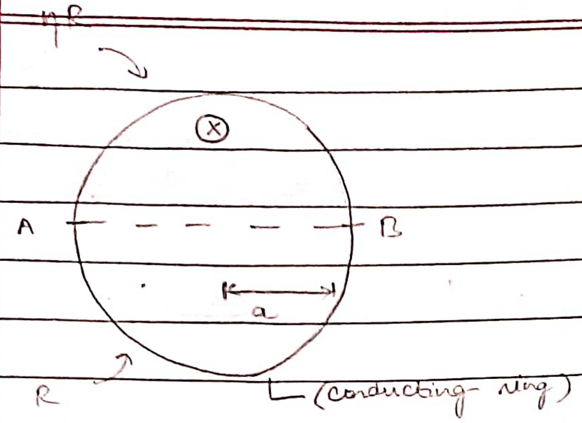
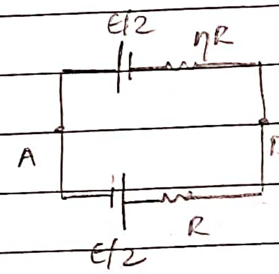
Q.



$$\frac{dB}{dt} = \alpha$$

Find $V_A - V_B$

A. $\mathcal{E} = \pi a^2 \alpha$ $\Rightarrow$



This is induced in the whole ring

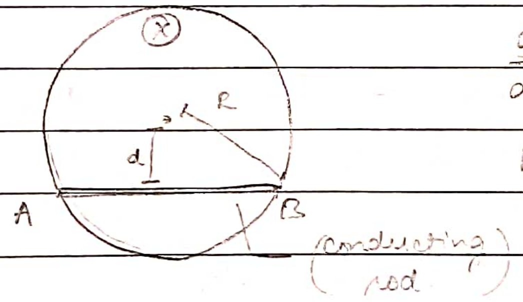
It is divided uniformly along length of ring.

Since $\overset{\frown}{AB} = \underset{\smile}{AB}$ $\Rightarrow$ $\mathcal{E}$ is equally divided

$$\mathcal{E}_{eq} = \frac{\frac{\mathcal{E}}{2\eta R} - \frac{\mathcal{E}}{2R}}{\frac{1}{\eta R} + \frac{1}{R}} = \left(\frac{\mathcal{E}}{2}\right)\left(\frac{1-\eta}{1+\eta}\right)$$

$$\Rightarrow \quad V_A - V_B = \left(\frac{\mathcal{E}}{2}\right)\left(\frac{1-\eta}{1+\eta}\right)$$

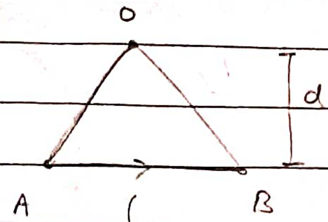
Q.



$$\frac{dB}{dt} = \alpha$$

Find $V_A - V_B$

A. We assume a conducting loop



(current in this dirn is as to opp. $\frac{dB}{dt}$)

Since $E_i \perp$ OA & OB.

$\Rightarrow$ $\epsilon_{induced}$ in OA & OB is ZERO

So, all emf is induced across AB.

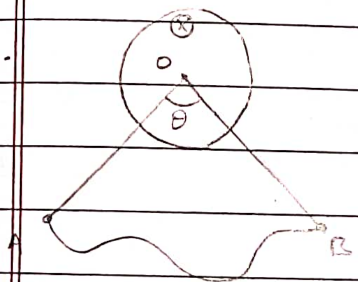
$\epsilon = \frac{1}{2}(d)(2\sqrt{R^2-d^2})\,\alpha \Rightarrow \epsilon_{AB} = -\alpha d\sqrt{R^2-d^2}$

(since pot. ↑ in dirn of induced)

Alternate Method

$V_{AB} = \oint \vec{E_i}\cdot\vec{dl}$

Q.



$\frac{dB}{dt} = \alpha$

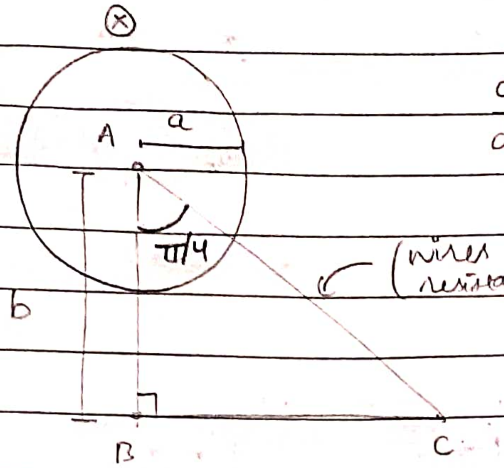
Find $V_A - V_B$

A. By the argument we used in above Q,

$\epsilon_{induced}$ in OA or OB is ZERO., so all emf is induced across AB

$\epsilon = \left(\frac{1}{2}\theta a^2\right)\alpha \Rightarrow \epsilon_{AB} = -\frac{\theta a^2 \alpha}{2}$

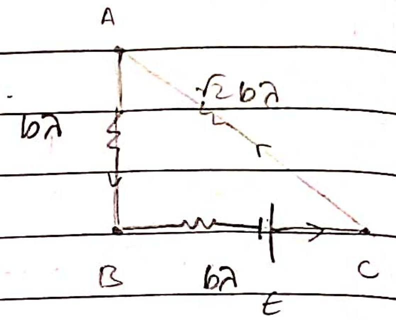
Q.



$\frac{dB}{dt} = \alpha$

Find $V_A - V_B$ & $V_B - V_C$

(wires of resistance λ Ω/m)



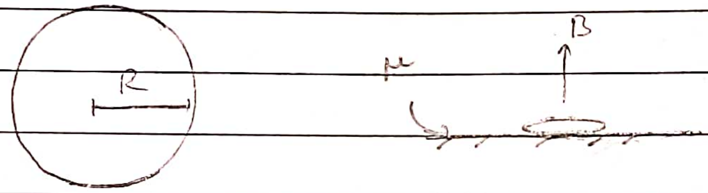
A. $\epsilon = \frac{(\pi/4)\, a^2}{2} = \left(\frac{\pi a^2}{8}\right)$

$i = \frac{\epsilon}{(2+\sqrt{2})\, b\lambda}$ $\Rightarrow$ $V_B - b\lambda i + \epsilon = V_C$

$\Rightarrow$ $\underline{V_B - V_C = b\lambda i - \epsilon}$

$\underline{V_A - V_B = b\lambda i}$

Q.



(m, q) uniform

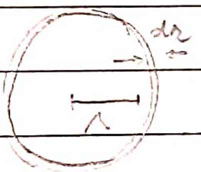
At $t = 0$, a vertical B co-axial with conducting disc is switched on

$B = B_0 t^2$ T

a) Find time 't_0' when the disc will start rotating

b) Find angular momentum of disc at $t = 2t_0$

A. a)



$(E_i)(2\pi r) = (\pi r^2)(2B_0 t)$

$\Rightarrow$ $E_i = B_0 r t$

$d\tau_{E_i} = dq\, E_i\, r = 2\pi \sigma_q B_0 r^3 t\, dr$

$d\tau_\mu = \mu g\, dm\, r = 2\pi \sigma_m \mu g r^2\, dr$

Disc starts rotating when $\underline{\tau_{Ei} = \tau_{\mu}}$

$$\Rightarrow \int_0^R \cancel{2\pi}\sigma_q B_0 t_0 r^3 \, dr = \int_0^R \cancel{2\pi}\sigma_m \mu g r^2 \, dr$$

$$\Rightarrow \sigma_q B_0 t_0 \left(\frac{R^4}{4}\right) = \frac{\sigma_m \mu g R^3}{3}$$

$$\Rightarrow t_0 = \left(\frac{\sigma_m}{\sigma_q}\right)\left(\frac{4\mu g}{3B_0 R}\right) = \underline{\frac{4m\mu g}{3qB_0 R}}$$

b) $$\tau = \tau_{Ei} - \tau_{\mu} = \frac{qB_0R^2}{2} t - \frac{2}{3}\mu m g R$$

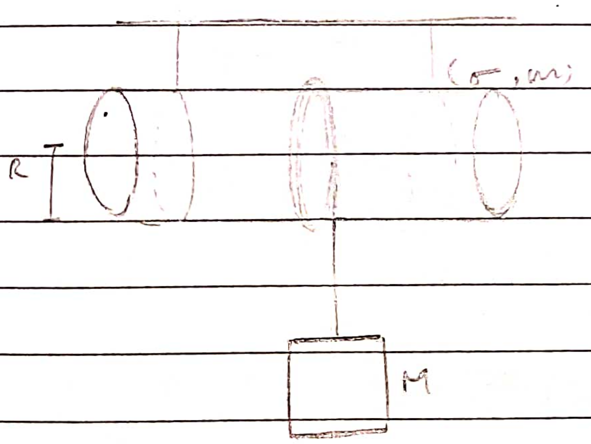
$$\Rightarrow \left(\frac{mR^2}{2}\right)\left(\frac{d\omega}{dt}\right) = \frac{qB_0R^2}{2} t - \frac{2}{3}\mu m g R$$

$$\Rightarrow \int_0^{\omega} d\omega = \int_{t_0}^{2t_0} \frac{qB_0}{m} t - \frac{4\mu g}{3R} \, dt$$

(Rot^n starts after $t = t_0$)

$$\Rightarrow \omega = \left(\frac{qB_0}{m}\right)\left(\frac{3t_0^2}{2}\right) - \frac{4\mu g t_0}{3R}$$

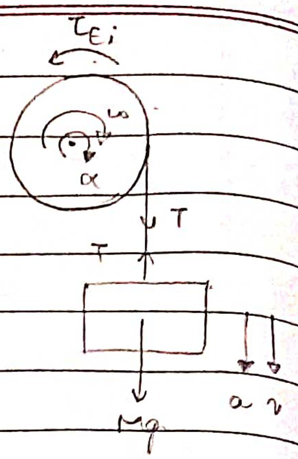
☆Q.



Find ω as a fn^n of time.

A. q rotates → i created

B created ← (cylinder acts like solenoid) ←



$\omega \uparrow \rightarrow i \uparrow \Rightarrow B \uparrow \Rightarrow \frac{d\phi}{dt} \uparrow$

(Provides torque) ← E_i created ←

$$B = \mu_0 \underbrace{ni}_{\text{(current/length)}} = (\mu_0)\left(\frac{Ni}{L}\right) = \left(\frac{\mu_0}{L}\right)\left(\frac{q}{2\pi/\omega}\right) = \left(\frac{\mu_0}{L}\right)\left(\frac{\sigma \cdot \cancel{2\pi} RL}{\cancel{2\pi}/\omega}\right)$$

$$= \mu_0 \sigma R \omega$$

$$E_i (2\pi R) = BA = (\mu_0 \sigma R \omega)(\pi R^2)$$

$$\Rightarrow E_i = \left(\frac{\mu_0 \sigma R^2}{2}\right)\alpha$$

$$\tau = I\alpha \Rightarrow (mR^2)\alpha = TR - \int E \cdot R \cdot dq$$

$$\Rightarrow mR^2\alpha = TR - ER\sigma 2\pi RL$$

Also, $Mg - T = Ma = M\alpha R \Rightarrow T = Mg - MR\alpha$

$$\Rightarrow mR^2\alpha = MgR - MR^2\alpha - \mu_0\pi R^3 L\sigma^2\alpha$$

$$\Rightarrow \alpha = \frac{MgR}{mR^2 + MR^2 + \mu_0\sigma\pi R^3 L\sigma^2}$$

Since α = const. $\Rightarrow v = at = R\alpha t$

$$\Rightarrow v = \left(\frac{Mg}{m + M + \mu_0\pi RL\sigma^2}\right) t$$

10/08/2023

INDUCTANCE

Resistor → opposes current flow
Capacitor → opposes voltage change
Inductor → opposes magnetic flux change

Inductance is a geometrical ppt. i.e does not depend on material of inductor.

When current is passed through a cond. loop, φ through it changes.
$\Delta\varphi$ induces an emf across the loop

This ϵ_i causes a current to flow in the loop. This phenomenon is known as self - inductance.

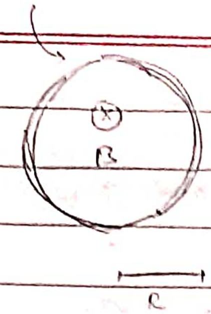
$$\boxed{\epsilon_i = -\frac{d\varphi_L}{dt} = -L\frac{di}{dt}}$$

(Coeff of self inductance)

SI unit: 1 Henry
or 1 Wb/A (Weber/A).
or 1 T-m²/A

NOTE: φ that we use while determining ϵ_i, is not magnetic flux, rather it is linked magnetic flux.
We shall denote it by φ_L

n-coils

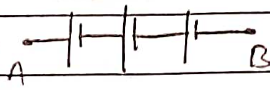


eg - (i) **Ring**

$$\varphi = B.A = \left(\frac{n\mu_0 i}{2R}\right)(\pi R^2)$$

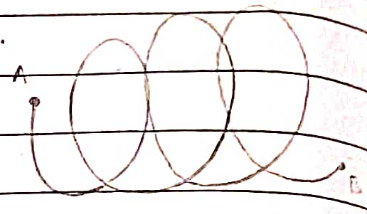
Consider the emf induced in each coil

$$\varepsilon_{i_{Total}} = n\,\varepsilon_{i_{\substack{each\\coil}}} \Leftarrow$$



(series combination of batteries)

$\equiv$



$$= -n\left(\frac{d\varphi}{dt}\right)$$

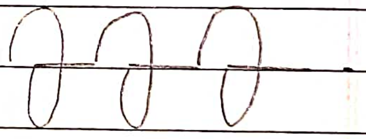
Hence, we define φ_L s.t $\varepsilon_{i_{Total}} = \frac{-d\varphi_L}{dt}$

$$\Rightarrow \frac{-d\varphi_L}{dt} = \frac{-n\,d\varphi}{dt} \Rightarrow \varphi_L = n\varphi$$

$$\frac{L\,di}{dt} = \frac{d\varphi_L}{dt} \Rightarrow L_{Ring} = \frac{d\varphi_L}{di} = \frac{B(nA)}{i} = \boxed{\frac{n^2\mu_0\pi R}{2}}$$

(# Loops)

(ii) **Solenoid**



$\varphi_L = n\varphi$

$$\Rightarrow L_{solenoid} = n\left(\frac{d\varphi}{di}\right)$$

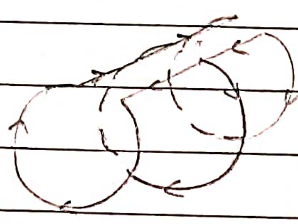
$\equiv$

$$\Rightarrow \boxed{L_{solenoid} = \pi R^2 \mu_0 n^2 l}$$

(# turns/length)

NOTE:

$$\varepsilon_{net} = \varepsilon/n = \frac{1}{n}\left(\frac{d\varphi}{dt}\right) = \frac{d}{dt}\left(\frac{\varphi}{n}\right)$$



$$\Rightarrow \boxed{\varphi_L = \frac{\varphi}{n}}$$

$\equiv$

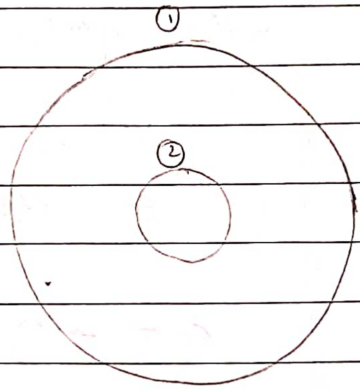
→ Mutual Inductance

If two loops are placed nearby & current is passed through one of them, $\Delta\varphi$ induces ϵ_i in the other.
This phenomenon is known as mutual inductance

$$\frac{d\varphi_2}{dt} = M_{12}\frac{di_1}{dt}$$

$$\frac{d\varphi_1}{dt} = M_{21}\frac{di_2}{dt}$$

(current change) (linked flux change)



By Reciprocal Thm, $\boxed{M_{21} = M_{12}}$

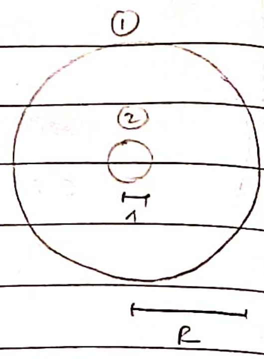
$\Downarrow$

φ_1	=	φ_2
when i through ②		when i through ①

To calc. M or φ, we choose which loop to pass current through as per our convenience.

Q. 2 concentric coplanar rings have radii r & R (r << R). If i is passed through ring of radius 'r', find φ linked with ring of radius R.

A. φ_L through ① cannot be calc. directly as B is varying with space.



We instead pass i through ① to calc. ε_i in ②

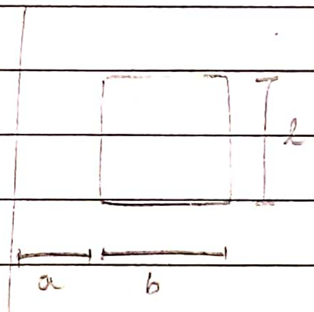
$$\varepsilon_i = -M_{21}\frac{di}{dt} = \frac{-d\varphi_{L_2}}{dt}$$

By Reciprocity Thm, $M_{21} = M_{12}$

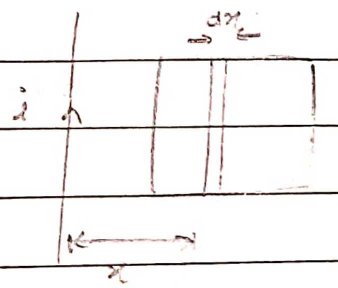
$$\Rightarrow \frac{\varphi_{L_2}}{i} = \frac{\varphi_{L_1}}{i}$$

$$\Rightarrow \varphi_{L_1} = \left(\frac{\mu_0 i}{2R}\right)(\pi r^2)$$ [assuming B const as r << R]

Q. Find M.I



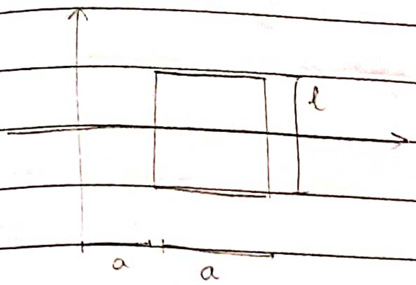
A.



$$d\varphi_L = \left(\frac{\mu_0 i}{2\pi x}\right)(l\,dx)$$

$$\Rightarrow \frac{\varphi_L}{i} = \left(\frac{\mu_0 l}{2\pi}\right)\int_a^{(b+a)}\frac{dx}{x}$$

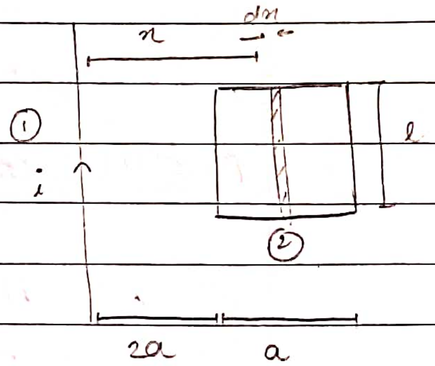
$$\Rightarrow \boxed{M = \frac{\mu_0 l}{2\pi}\ln\left|1+\frac{b}{a}\right|}$$

Q. 

Find φ in x-y plane for $x \le -a$

A. We assume an ∞ wire at $x = -a$

By Reciprocity Thm,

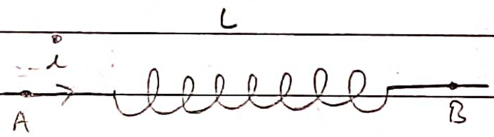


$$\varphi_1 = \varphi_2 = \int d\varphi_2$$

$$\Rightarrow \varphi_1 = \int_{2a}^{3a} \left(\frac{\mu_0 i}{2\pi x}\right)(l\, dx) = \frac{\mu_0 i l}{2\pi} \ln\left(\frac{3}{2}\right)$$

→ Inductor in circuit

$\varphi = Li$

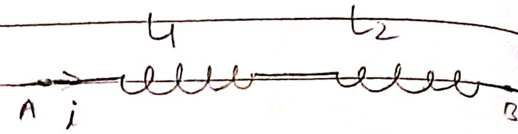


$\mathcal{E} = -L \frac{di}{dt}$

(Electronic symbol for inductor)

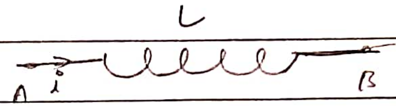
So, $V_A - L \frac{di}{dt} = V_B$

- **Series comb.** –



$$V_A - L_1\frac{di}{dt} - L_2\frac{di}{dt} = V_B$$

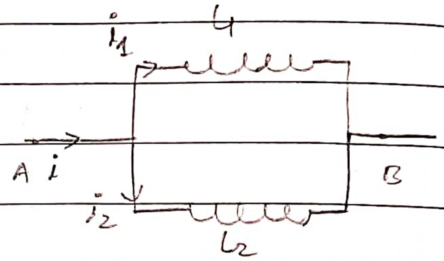
≡



$$V_A - L\frac{di}{dt} = V_B$$

$$\Rightarrow \boxed{L = L_1 + L_2}$$

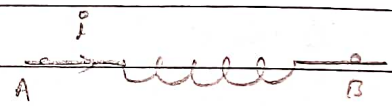
- **Parallel comb.** –



$$i = i_1 + i_2$$

$$\Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

≡

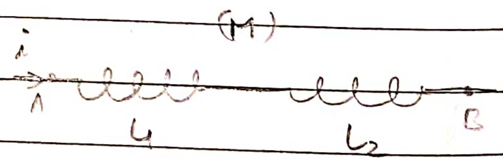


$$V_A - V_B = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt} = L\frac{di}{dt}$$

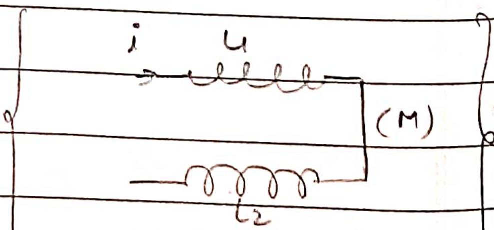
$$\Rightarrow \boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}}$$

- **Comb. of Ind. with M.I** –

series) $\varphi_1 = L_1 i + \underbrace{Mi}$ (if φ in same dirⁿ)



$$\varphi_2 = L_2 i + Mi$$



$$\varphi = L_1 i - Mi$$

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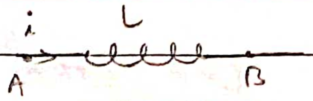
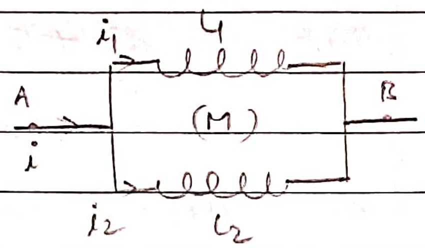
$$V_A - V_B = V_A - V_B \quad \equiv$$



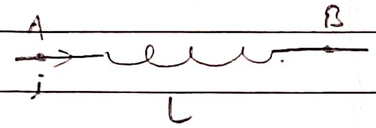
$$\Rightarrow \quad L\frac{di}{dt} = (L_1+M)\frac{di}{dt} + (L_2+M)\frac{di}{dt} \quad \Rightarrow \quad \boxed{L = L_1+L_2+2M}$$

(Parallel) $\quad i = i_1 + i_2$

$$\Rightarrow \quad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$



≡



$$V_A - V_B = V_A - V_B$$

$$\Rightarrow \quad L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \quad -(i)$$

$$= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \quad -(ii)$$

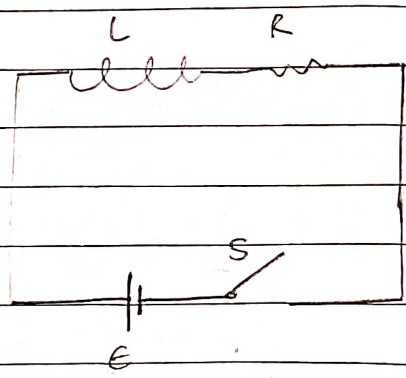
$$L_2(i) - M(ii) \quad \Rightarrow \quad \frac{di_1}{dt}(L_1L_2 - M^2) = L\frac{di}{dt}(L_2 - M)$$

similarly $$\frac{di_2}{dt}(L_1L_2 - M^2) = L\frac{di}{dt}(L_1 - M)$$

$$\Rightarrow \quad \frac{1}{L} = \frac{L(L_2-M)}{L_1L_2-M^2} + \frac{L(L_1-M)}{L_1L_2-M^2}$$

$$\boxed{\frac{1}{L} = \left(\frac{L_1+L_2-2M}{L_1L_2-M^2}\right)}$$

L-R CIRCUIT

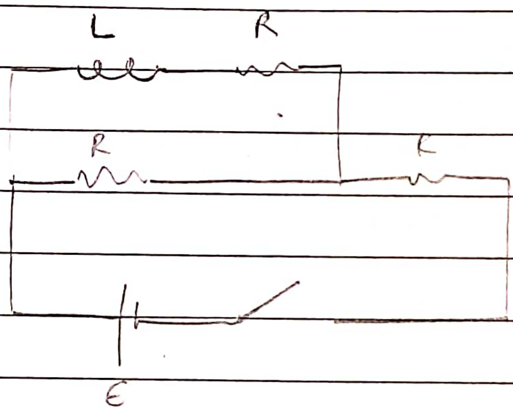


Just after closing the switch, current through inductor will not change.

But $\frac{di}{dt} \neq 0$

In steady state however, $\frac{di}{dt} = 0$

So inductor behaves as regular wire.

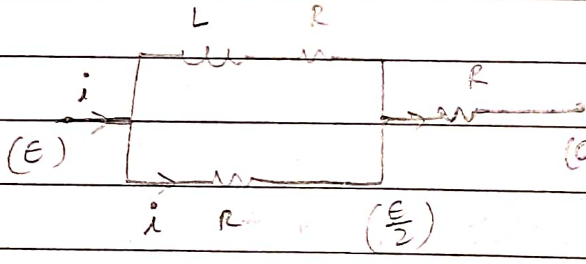
Q 

Switch closed at $t=0$. Find current through battery & V across ind.

a) just after closing switch

b) in steady state

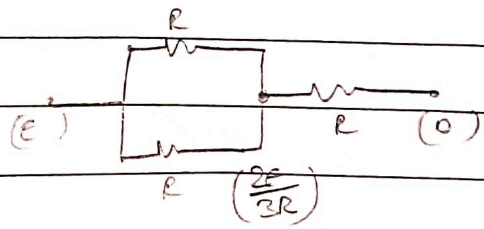
A. a)



$i = \frac{E}{2R}$

$V_R + V_L = \left(\frac{E}{2}\right) \Rightarrow \frac{di}{dt} = \frac{E}{2L}$

b)



$i = \frac{2E}{3R}$

$V_L = 0$

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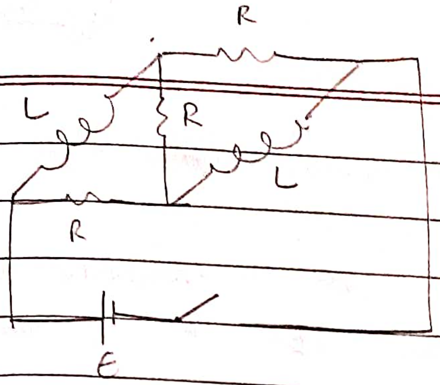
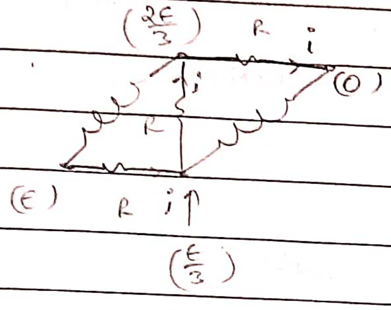
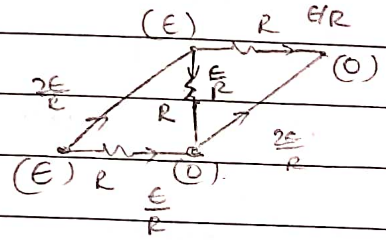
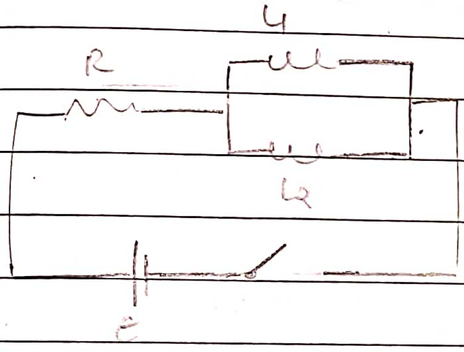
Q. 

A. a) 

$i = \frac{E}{3R} \Rightarrow V_L = \frac{E}{3}$

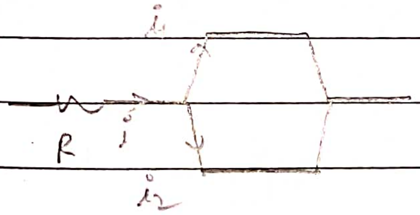
b) 

$i = \left(\frac{3E}{R}\right)$

Q. 

Find current through each indo. in steady state

A. $i = \frac{E}{R}$



since $V_{L_1} = V_{L_2}$

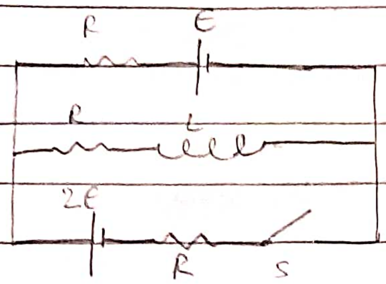
$\Rightarrow L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \quad \forall\ t$

$\Rightarrow L_1 i_1 = L_2 i_2$

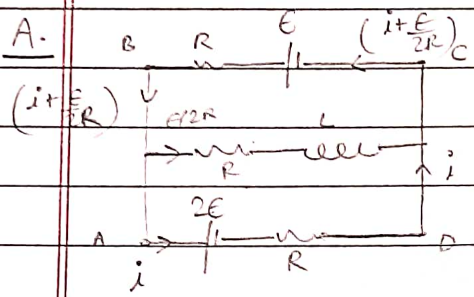
$\Rightarrow i_1 = \left(\frac{L_2}{L_1+L_2}\right)\left(\frac{E}{R}\right)$

$i_2 = \left(\frac{L_1}{L_1+L_2}\right)\left(\frac{E}{R}\right)$

17/08/2023

Q. 

At $t=0$, the switch is closed. Find the current through battery of 2ϵ & V across the ind. just after closing the switch

A. 

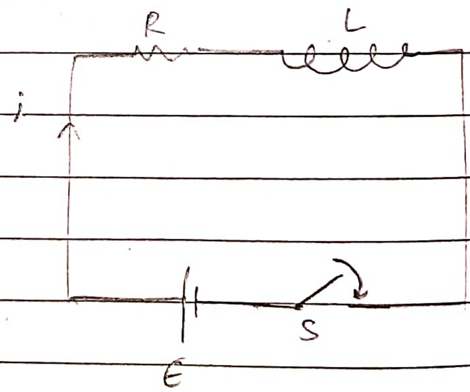
Initially $i = \frac{\epsilon}{2R}$ flowing through ind. (steady state)

When switch closed, no change in i through ind.

In ABCD, $2\epsilon + \left(i + \frac{\epsilon}{2R}\right)R - \epsilon + iR = 0$

$\Rightarrow \frac{3\epsilon}{2} = -2iR \Rightarrow i = \frac{-3\epsilon}{4R}$

→ Growth & decay of current



$$-Ri - L\frac{di}{dt} + \epsilon = 0$$

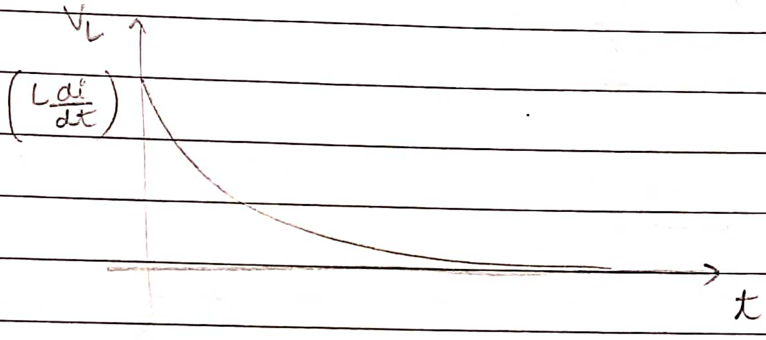
$$\Rightarrow L\frac{di}{dt} = \epsilon - Ri$$

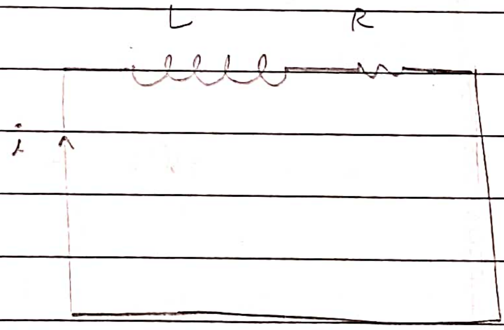
$$\Rightarrow \int_0^i \frac{di}{\epsilon - Ri} = \int_0^t \frac{dt}{L}$$

$$\Rightarrow \ln\left(\frac{\epsilon - Ri}{\epsilon}\right) = -\frac{Rt}{L}$$

$$\Rightarrow \boxed{i = \frac{\epsilon}{R}\left(1 - e^{-\frac{t}{LR}}\right)}$$

Time const (τ) : $\boxed{\tau = \frac{L}{R}}$





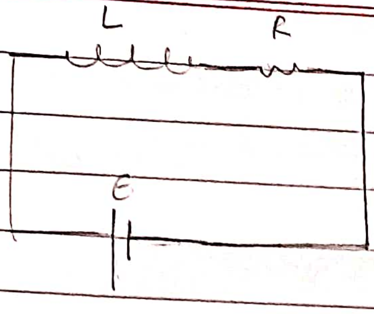
$$-L\frac{di}{dt} - Ri = 0$$

$$\Rightarrow \int_{i_0}^{i} \frac{di}{i} = -\int_{0}^{t} \frac{R}{L}\, dt$$

$$\Rightarrow l\left(\frac{i}{i_0}\right) = \frac{-Rt}{L}$$

$$\Rightarrow \boxed{i = i_0 e^{-\frac{t}{LR}}}$$

✯ Q

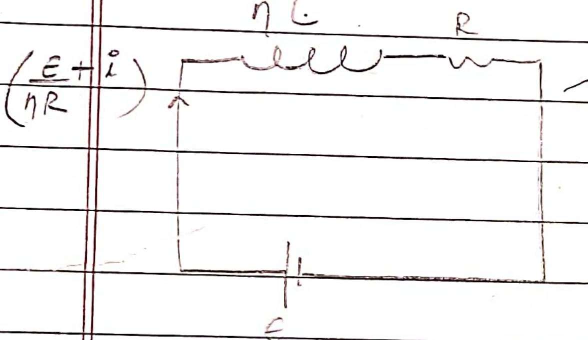


$L \to \eta L$ at $t = 0$
Find current in circuit as a fn^n of time

A. Initially $i_L = \left(\frac{E}{R}\right)$

Since ind. tries to resist $\Delta\varphi$, changing L suddenly changes i s.t $\Delta\varphi = 0$

$\Rightarrow \quad i' = \left(\frac{E}{\eta R}\right)$



$$-\eta L \frac{di}{dt} - R\left(\frac{E}{\eta R} + i\right) + E = 0$$

$$\Rightarrow \eta L \frac{di}{dt} + Ri = E\left(1 + \frac{1}{\eta}\right)$$

$$\Rightarrow i = \frac{E}{R}\left(1 + \frac{1}{\eta}\right)\left(1 - e^{\frac{-Rt}{\eta L}}\right)$$

$$i(t) = i + \frac{E}{\eta R} = \left(\frac{E}{R}\right)\left(\frac{1}{\eta} + \left(1 + \frac{1}{\eta}\right)\left(1 - e^{\frac{-Rt}{\eta L}}\right)\right)$$

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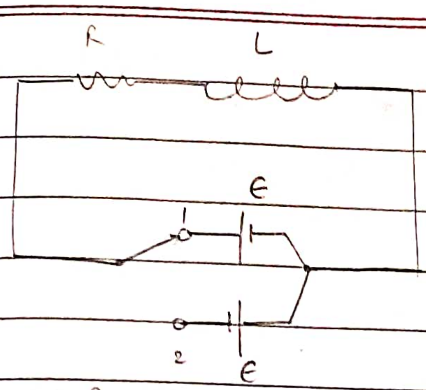
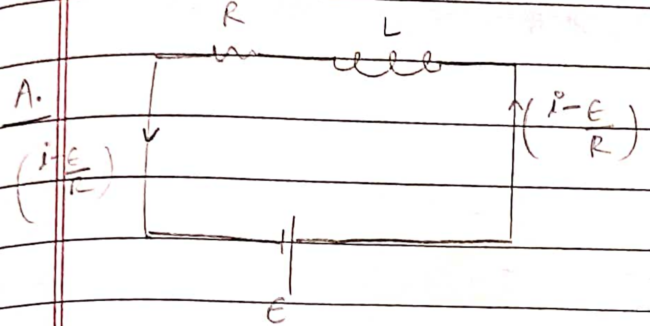
Q. 

At $t=0$, switch moved from port 1 to 2
Find $i(t)$

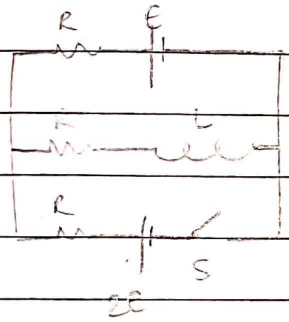
A. 

$$-L\frac{di}{dt} - R\left(i-\frac{E}{R}\right) + E = 0$$

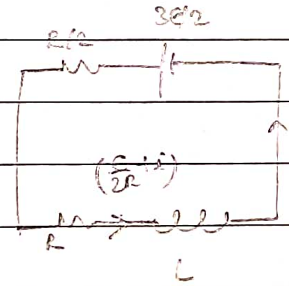
$$\Rightarrow L\frac{di}{dt} + Ri = 2E$$

$$\Rightarrow i = \left(\frac{2E}{R}\right)\left(1-e^{-\frac{t}{4R}}\right)$$

$$i(t) = i-\frac{E}{R} = \left(\frac{E}{R}\right)\left(1-2e^{-\frac{Rt}{L}}\right)$$

Q. 

Switch closed at $t=0$.
Find $i_L(t)$

A. $\equiv$ 

$$-\frac{3R}{2}\left(\frac{E}{2R}+i\right) - L\frac{di}{dt} + \frac{3E}{2} = 0$$

$$\Rightarrow L\frac{di}{dt} + \left(\frac{3R}{2}\right)i = +\frac{3E}{4}$$

$$\Rightarrow i = \left(\frac{+3E}{4}\right)\left(\frac{2}{3R}\right)\left(1-e^{-\frac{3tR}{2L}}\right)$$

$$= \frac{E}{2R}\left(1-e^{-\frac{3tR}{2L}}\right)$$

$$\Rightarrow i(t) = i+\frac{E}{2R} = \left(\frac{E}{2R}\right)\left(2-e^{-\frac{2Rt}{2L}}\right)$$

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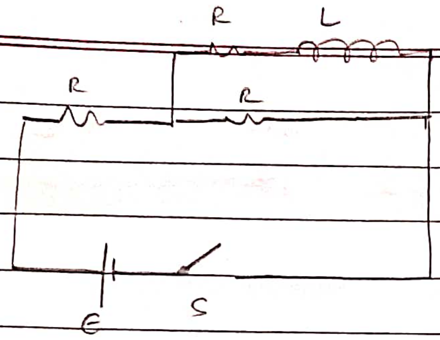
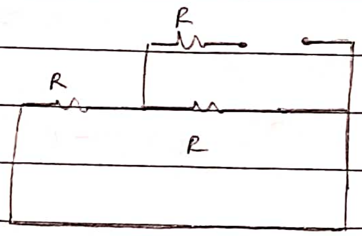
☆ Q



Find $i_L(t)$

A. To find τ, consider R_{eff} of circuit abt L



$\Rightarrow R_{eff} = 3R/2$

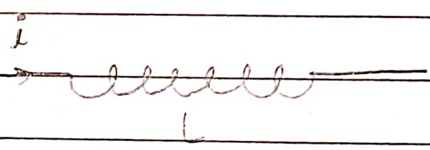
$\Rightarrow \tau = \frac{2L}{3R}$

Steady state current through L is $\left(\frac{E}{3R}\right)$

$$\Rightarrow i_L(t) = \left(\frac{E}{3R}\right)\left(1 - e^{-\frac{3Rt}{2L}}\right)$$

→ Energy of Inductor

$V_L = L\frac{di}{dt}$



$dW = V_L\, dq = L\frac{di}{dt}dq = Li\,di$

$$\Rightarrow W = \int_0^{i_0} Li\,di = \frac{1}{2}Li_0^2 = \frac{\varphi^2}{2L}$$

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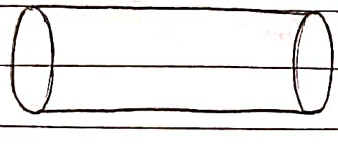
$L = \mu_0 n^2 \pi R^2 l$

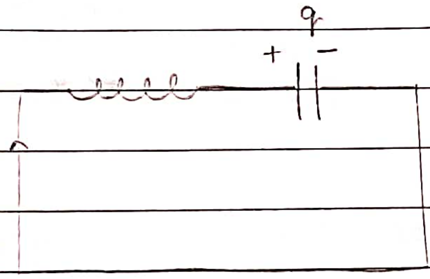

solenoid

$U = \frac{1}{2} L i^2$

$u_B = \frac{U}{Vol} = \frac{\mu_0 n^2 \cancel{\pi R^2 l}\, i^2}{2\,\cancel{(\pi R^2 l)}}$

(energy density of $\vec{B}$) $= \frac{B^2}{2\mu_0}$

L–C OSCILLATION



i charges q till

$i \to 0$.

When $i = 0$, $V_c = \frac{q}{C}$

$\Rightarrow V_L = \frac{q}{C} \Rightarrow \frac{di}{dt} = -\frac{q}{LC} \Rightarrow$ current starts flowing in opp. dir^n

So cap. starts discharging

Let us consider an L–C circuit with intial current i_0 & charge on cap. q_0
(charging current)

$L\frac{di}{dt} + \frac{q}{C} = 0 \Rightarrow L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$

$\Rightarrow \frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right) q$

This is eq^n of SHM $\Rightarrow$ Charge performs SHM!

So $\quad q = q_{max}\, s(\omega t + \varphi)$, $\quad \boxed{\omega = \frac{1}{\sqrt{LC}}}$

$$\frac{dq}{dt} = q_{max}\, \omega\, c(\omega t + \varphi)$$

given $\quad q_0 = q_{max}\, s(0+\varphi)$

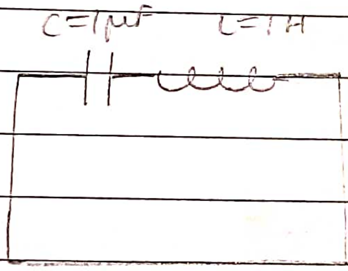
$\quad i_0 = q_{max}\, \omega\, c(0+\varphi)$

$$\Rightarrow \frac{q_0}{i_0} = t\varphi \Rightarrow \boxed{\varphi = t^{-1}\left(\frac{q_0}{i_0}\right)}$$

$$\boxed{q_{max} = \sqrt{q_0^2 + \left(\frac{i_0}{\omega}\right)^2}}$$

Therefore, $\quad q = \sqrt{q_0^2 + \left(\frac{i_0}{\omega}\right)^2}\; s\left(\frac{t}{\sqrt{LC}} + t^{-1}\left(\frac{q_0}{i_0}\right)\right)$

Q.



at $t=0$, $q_0 = \sqrt{3}\,\mu C$

& $i_0 = 1mA$

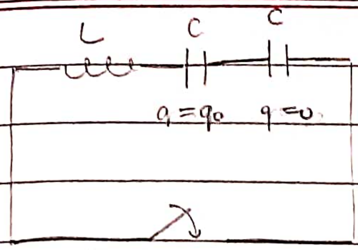
Find $q(t)$ & t_0 s.t $i(t_0) = 0$

A. $\omega = 10^3 \Rightarrow q_{max} = \sqrt{3+1} = 2\,\mu C$, $\varphi = t^{-1}\left(\frac{\sqrt{3}}{1}\right) = \left(\frac{\pi}{3}\right)$

$$q = 2\sin\left(10^3 t + \pi/3\right)\ \mu C$$

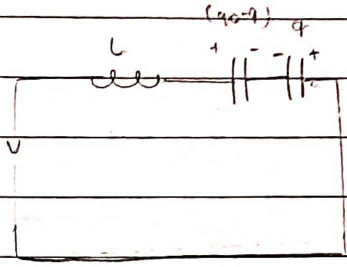
When $i(t) = 0 \Rightarrow q = q_{max} \Rightarrow 10^3 t + \left(\frac{\pi}{3}\right) = \left(\frac{\pi}{2}\right)$

$$\Rightarrow t = \frac{\pi}{6} \times 10^{-3}$$

Q. 

Find charge on uncharged cap. as a f^{xn} of time.

A. $L\frac{di}{dt} - \frac{(q_0 - q)}{C} + \frac{q}{C} = 0$



$\Rightarrow \frac{2q}{C} - \frac{q_0}{C} = -L\frac{di}{dt} = -L\frac{d^2q}{dt^2}$

☆ $\Rightarrow \frac{d^2}{dt^2}\left(q - \frac{q_0}{2}\right) = -\frac{2}{LC}\left(q - \frac{q_0}{2}\right)$

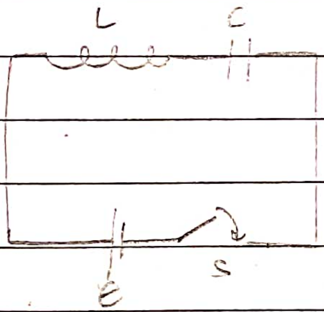
$\Rightarrow q - \frac{q_0}{2} = A\,s(\omega t + \varphi)$; $\omega = \sqrt{\frac{2}{LC}}$

$\Rightarrow i = A\omega\, c(\omega t + \varphi)$

$i_0 = 0 \Rightarrow \varphi = \left(\frac{\pi}{2}\right) \Rightarrow q = \frac{q_0}{2} + A\,s(\omega t + \pi/2)$

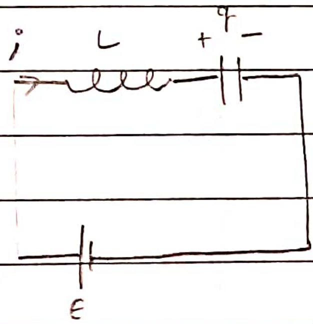
$q(0) = 0 \Rightarrow 0 = \frac{q_0}{2} + A \Rightarrow A = -q_0/2$

$\Rightarrow q = \frac{q_0}{2}(1 - C\omega t)$

Q 

Find $q(t)$ & $i(t)$

A. $E - L\frac{di}{dt} - \frac{q}{C} = 0$



$\Rightarrow \frac{1}{C}(q - EC) = -L\frac{d^2q}{dt^2}$

$$\Rightarrow \frac{d^2}{dt^2}(q-\epsilon C) = -\frac{1}{LC}(q-\epsilon C)$$

$$\Rightarrow q-\epsilon C = A \sin(\omega t+\varphi) \quad , \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow i = A\omega \cos(\omega t+\varphi)$$

$i_0 = 0 \Rightarrow \varphi = \pi/2$

$q_0 = 0 \Rightarrow 0 - \epsilon C = A \Rightarrow \underline{A = -\epsilon C}$

$$\Rightarrow \underline{q = \epsilon C(1-\cos\omega t)}$$

$$\underline{i = \epsilon C\omega \sin\omega t}$$

→ Damped oscillation

(Damping const.)

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

$$\Rightarrow \boxed{y = A_0 e^{-\left(\frac{b}{2m}\right)t} \sin(\omega' t+\varphi)}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$\underline{A = A_0 e^{-\left(\frac{b}{2m}\right)t}}$$

(Max. ampo)

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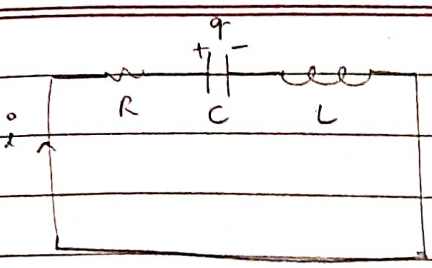


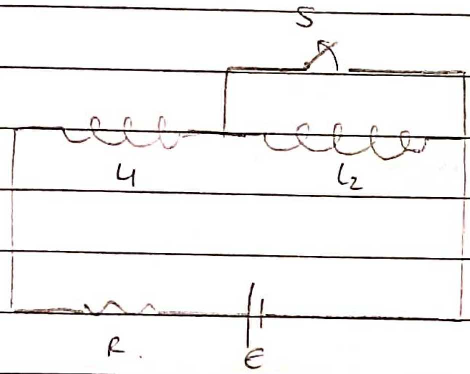
$$-iR - \frac{q}{C} - L\frac{di}{dt} = 0$$

$$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\Rightarrow q = q_0 e^{\left(-\frac{Rt}{2L}\right)} \sin(\omega' t + \varphi)$$

$$q_{max} = q_0 e^{\left(-\frac{Rt}{2L}\right)}$$

(max. max. charge)

Q. 

Find current immediately after opening S. & in new steady state

A. $i = \left(\frac{E}{R}\right)$

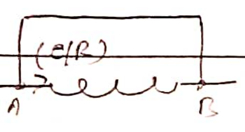
Ind. changes current to conserve flux of circuit.

$$\Rightarrow (L_1+L_2)(i') = L_1 i$$

$$\Rightarrow i' = \left(\frac{L_1}{L_1+L_2}\right)\left(\frac{E}{R}\right)$$

$i_s = \left(\frac{E}{R}\right)$

* Q. In the above Q, if S closed again, find i, just after closing the switch

A. 

$V_{AB} = 0$
$V_{AB}' = 0$ (after closing switch)
$\Rightarrow$ No current through wire $\Rightarrow$ No flux change in inductor! $\Rightarrow$ same i as in steady state